Reservoir flood routing using one-dimensional flow model through rockfill dams

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Abstract
For the sustainable management of watersheds and flood control ,a model that can simulate the hydraulics of flow through rockfill dams is valuable. This paper describes a model of flow through rockfill dams. In the developed model an exponential relationship between Reynold's number (Re) and Darcy-Weisbach coefficient (*f*) is suggested. Using real field data and a nonlinear optimization technique, the relationship coefficients are obtained. By introducing inflow hydrograph and rockfill characteristics as an input data and utilizing the above relationship with one dimensional continuity equation, flow rating curve of rockfill dam can be identified and employed in a storage flood routing model. Outflow hydrograph of rockfill dam is the objective of the developed model. The accuracy of the numerical solution has been shown to be reliable wen compared to an exact analytical solution. The parametric sensitivity analysis demonstrated that the larger the rock, the less sensitive the water surface elevation will be.

Key words: Flood routing, Rockfill dam

Introduction

When rock is available, rockfill dam is an economical and fast tool for flood detention and control purposes. Rockfill dam can be designed satisfactorily when hydraulics of flow through rockfill dam is known. As this type of dam consists of coarse particles, the flow will deviate from Darcy's law and mostly is turbulent. This means that the relationship between the flow velocity, *V*, and its hydraulic gradient, *i*, is a nonlinear one. Different researchers proposed the following nonlinear relationships:

$$i = AV^{B} \tag{1}$$

$$i = A'V + B'V^{2} \tag{2}$$

where A, A', B, B' are coefficients dependent on the rock and fluid characteristics. Equation (1) was proposed by Prony in 1804 and Equation (2) by Forcheimer in 1901 (Li *et al*., 1998). Other researchers, to explain the hydraulics of flow through rocks, suggested relationships between Reynold's number (Re) and Darcy-Weisbach coefficient (*f*) in the following forms:

$$f = a\,\mathrm{Re}^{b} \tag{3}$$

$$f = \frac{a'}{\mathrm{Re}} + b' \tag{4}$$

where a, a', b, b' are also coefficients dependent on the rock and fluid characteristics. If the Reynold's number is written in terms of *v*, the Darcy-Weisbach can be expressed in the form of Equations (1) and (2), respectively. The following shows some of the relationships proposed by different researchers.

In 1952, Ergun proposed the following relationship (Ergun, 1952):

$$i = 150\frac{(1-n)^{2}\nu}{gn^{3}d^{2}}V + 1..75\frac{(1-n)}{dgn^{3}}V^{2} \tag{5}$$

where n = rock porosity, ν = kinematic viscosity, d = average size of rock material and g = acceleration due to gravity. Wilkins introduced the following relationship(Wilkins, 1956):

$$i = \frac{0.0465}{R_h^{0.925} n^{1.85}} V^{1.85} \quad (6)$$

where R_h is the hydraulic radius of the rock material. He considered R_h equal to $d/10$ for the range $0.4 \le d \ge 2$ meters. Ward used the following (Ward, 1964):

$$i = \frac{\nu}{gk} V + \frac{c_w}{g\sqrt{k}} V^2 \quad (7)$$

where c_w = a coefficient and k = intrinsic permeability. Leps introduced the following (Leps, 1973):

$$v = k_o R_h^{0.5} i^{0.54} \quad (8)$$

where k_o is a coefficient. McCorquodal used the following relationship (McCorquodal *et al.*, 1978):

$$i = \frac{70\nu}{gnR_h^2} V + \frac{0.81}{gn^{0.5} R_h} V^2 \quad (9)$$

Stephensen proposed a relationship in a form of (Stephenson, 1979):

$$f = \frac{800}{\mathrm{Re}} + k_t \quad (10)$$

where $\mathrm{Re} = \frac{Vd}{n\nu}$. k_t for a laminar flow through rock material is equal to zero and it takes different values when the rock shape changes. For turbulent flow ($\mathrm{Re} > 10000$), f is almost equal to k_t. Herrera and Felton introduced the following (Herrera and Felton, 1991):

$$f = \frac{3858}{\mathrm{Re}} + 17.6 \quad (11)$$

They defined Re as $\frac{V(d-\sigma)}{\nu}$ where σ is the standard deviation of rock size distribution. Li and his colleagues used the following relationship (Li *et al.*, 1998):

$$f = 8.75\,\mathrm{Re}^{-0.17} \quad (12)$$

in which d considered equal to $4R_h$. R_h and Re are defined as:

$$R_h = \frac{e}{A_{vs}} \quad (13) \qquad \mathrm{Re} = \frac{vR_h}{n\nu} \quad (14)$$

where e and A_{vs} are the void ratio and particle surface area per unit volume, respectively. According to the above review of the different relationships it is concluded that the hydraulic gradient may be defined from Darcy-Weisbach equation considering f in a form similar to Equation (3).

Materials and Methods

Model Development

To develop the model, either of Equations (3) or (4) can be combined with Darcy-Weisbach equation. When Equation (3) is used, $(d-\sigma)$ instead of d, and $i = \frac{\Delta h}{L}$ where L is the base length of the rockfill dam, Darcy-Weisbach equation becomes

$$i = \frac{a\,\mathrm{Re}^b}{(d-\sigma)}\frac{V^2}{2g} \tag{15}$$

If Re is defined as $\frac{V(d-\sigma)}{\nu}$ Equation (15) becomes

$$V = \alpha\ i^{\frac{1}{b+2}} \tag{16}$$

where

$$\alpha = \left(\frac{2g\nu^b}{a(d-\sigma)^{b-1}}\right)^{\frac{1}{b+2}} \tag{17}$$

Equation (16) is similar to Darcy's law except *i* exponent is not equal to unity. Combining Equation (16) with the continuity equation and defining *i* as $-\frac{dh}{dx}$, yields

$$Q = \alpha\left(-\frac{dh}{dx}\right)^{\frac{1}{b+2}} h.w \tag{18}$$ where

Q = flow rate,
h = water level inside the rockfill dam,
w= width of flow cross section,
x = the longitudinal coordinate in flow direction.
Integrating Equation (18) between the limits H_2 to H_1 for h and zero to D for x, the following equation is resulted:

$$Q = \left(\frac{1}{D}\right)^{\frac{1}{b+1}} \frac{\alpha w}{(3+b)^{\frac{1}{b+2}}}\left(H_1^{3+b} - H_2^{3+b}\right)^{\frac{1}{b+2}} \tag{19}$$

where D is defined according to Sharma (1979):

$$D = L - 0.7S_1 = L - 0.7H_1 \cot\beta_1 \tag{20}$$

β_1 is the angle of the upstream face of the dam with the horizontal direction, and H_2 and H_1 are the downstream and upstream water depths, respectively.

Similarly, if Equation (4) is considered instead of Equation (3), the following should be obtained,

$$Q^2 = \frac{M_1}{2M_2}\left(H_1^2 - H_2^2\right) - \frac{b'M_1}{M_2^2}\left(H_1 - H_2\right) + \frac{b'^2 M_1}{M_2^3}\ln\left|\frac{M_2H_1 + b'}{M_2H_2 + b'}\right| \tag{21}$$

where $M_1 = \frac{2g(d-\sigma)w^2}{D}$ and $M_2 = \frac{a'w\nu}{Q(d-\sigma)}$. Therefore, Equations (19) and (21) can be used as flow rating equations for one dimensional flow through rockfill dams.

Reservoir Flood Routing

In this investigation, it is assumed that the flow upstream the rockfill dam has no significant velocity and it is acting as a reservoir and the basic equation for its flow routing is

$$Q_i - Q_o = \frac{dS}{dt} \tag{22}$$

where
Q_i = reservoir inflow rate,

Q_o = reservoir outflow rate,

$\frac{dS}{dt}$ = storage changes with respect to time.

The finite difference form of Equation (22) is going to be as:

$$\frac{Q_i^n + Q_i^{n+1}}{2} - \frac{Q_o^n + Q_o^{n+1}}{2} = \frac{S^{n+1} - S^n}{\Delta t} \quad (23)$$

where *n* and *n+1* indicate two following time steps with Δt difference. Q_o^{n+1} can be obtained from combining Equations (23) with (19) or (21).

Results

Model Calibration

To determine the different calculation goals of this investigation, a computer program is developed. The model parameters which are indicated as the coefficients of Equations (3) and (4) are required. They are determined by application of a nonlinear optimization process. In the optimization process, the coefficients are determined in such a way where the sum squares of the differences of the calculated outflows , Q_{oc}, and measured ones , Q_{om}, as an objective function is minimized. In this optimization process the objective function to be minimized is the difference between the calculated and measured outflow rates.

The experimental data used in this calibration was Hansen's data (Hansen *et al*., 1995). The data includes flow rates, upstream and downstream water levels for different rock sizes, 25 to 130 mm. The optimized result for Equation (2) was

$$f = 54\,\mathrm{Re}^{-0.077} \quad (24)$$

By substituting Equation (24) in Equation (16) yields:

$$V = 1.027(d_{50} - \sigma)^{0.56} i^{0.52} \quad (25)$$

where d_{50} is the average size of the rockfill material.

Model Validation

To check the accuracy of the model the following control steps were conducted:
Real field data were employed to check the validity of the model. In this regard, the data collected at the Jehad Sazendagi watersheds management research center in Tehran/Iran, were used. The data includes flow rates, upstream and downstream water levels for different rock sizes, 41 to 94 mm. As Tablet 1 indicates good agreement between the calculated and measured values. In addition, the results of the model were compared to other equations introduced by other researchers. Figure 1 has shown much more agreement with Wilkins' Equation results than the other equations.

An example of applications

In this regard, the model has been used to introduce flow rating curve and flood routed hydrograph for the following input data:
L=3 m, dam width, *w=5 m, ,* σ*=0,* d_{50}*=50 mm,* $\beta_1 = \beta_2 = 90^o$ where β_2 is the angle of the downstream face of the dam with the horizontal direction, (Mannings' coefficient) for downstream channel = *0.014*, slope of downstream channel = *0.001*, and the reservoir length = *2000* m.

Figures 2 and 3 show samples of the model flow rating curve and flood routing, respectively.

Conclusions

1. The optimized relationship, $f = 54\,\mathrm{Re}^{-0.077}$, is obtained for rockfill material in the range of 25 to 130 mm. For sizes larger than 130 mm, the above relationship is applicable with caution for two reasons. First, the above coefficients are comparable to Wilkins' coefficients where Wilkins' relationship is applicable for sizes up to 2 meters. Second, as Table 1 shows, the model sensitivity to large d_{50} is insignificant.
2. The model sensitivity to the different related parameters is different. Its sensitivity to *b* and *d* (smaller than 0.2 meter) more than the other parameters.
3. The model has the capability of prediction with good accuracy, the flow rating curve, outflow hydrograph for specific inflow hydrograph and rockfill characteristics (see Figure 1 and 2).
4. Determining the rockfill characteristics needed for the case of known inflow hydrograph and downstream flow rating curve.

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Table1-Model results versus experimental data

1-D	Observed	Observed							
H1(cm)	H1(cm)	H2(cm)	β_2	β_1	W(cm)	L(cm)	$\sigma(cm)$	d50(cm)	Q(l/s)
32	30.1	4.1	90	90	60	80	0	5.7	14.1
39.2	37.1	4.4	90	90	60	80	0	6.9	21.3
41.9	38.2	5.9	90	90	60	80	1	9.4	26.3
48.8	44.8	4.9	90	90	60	80	1	6.9	27.3

54.1	51.5	4.8	90	90	60	69	0	4.1	28.1
46.1	52.7	6.4	90	90	60	69	0	6.9	29.5
44.5	42.6	4.6	90	90	60	79	1	5.7	21
43.9	46.5	7.3	90	90	60	81.5	2	8.2	23.6
42.9	38.9	4.9	90	90	60	80	1	5.7	19.7
45.6	43.9	6.6	90	90	60	81	1	8.2	27.3

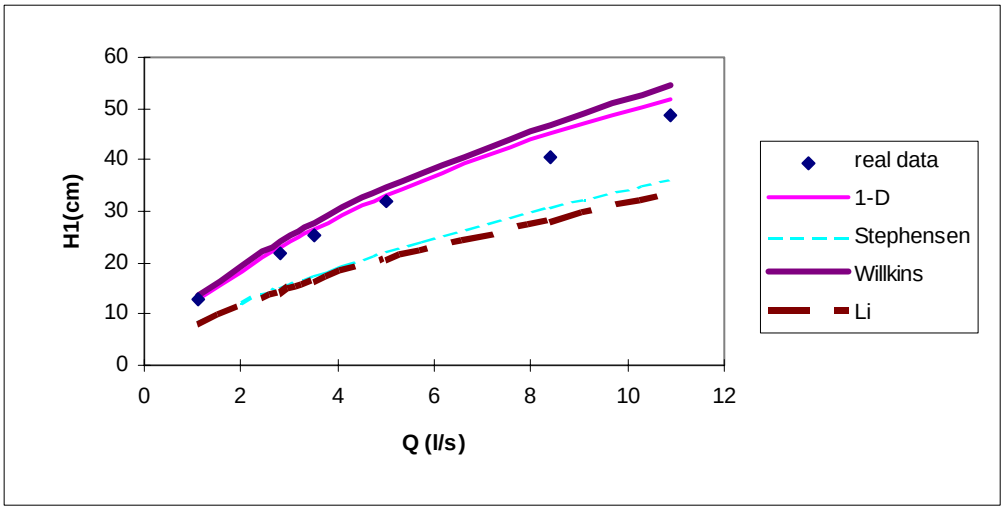

Fig.1-model and other reseacher's results

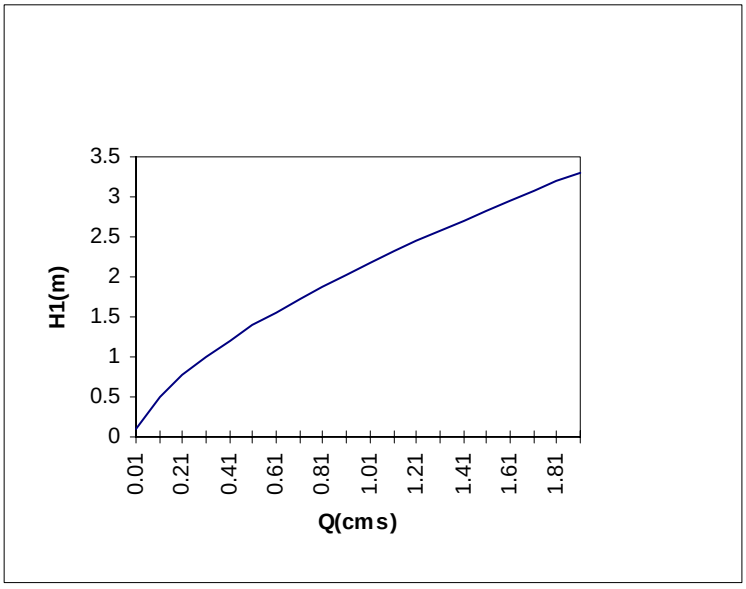

Fig.2-Rating curve calculated by the model

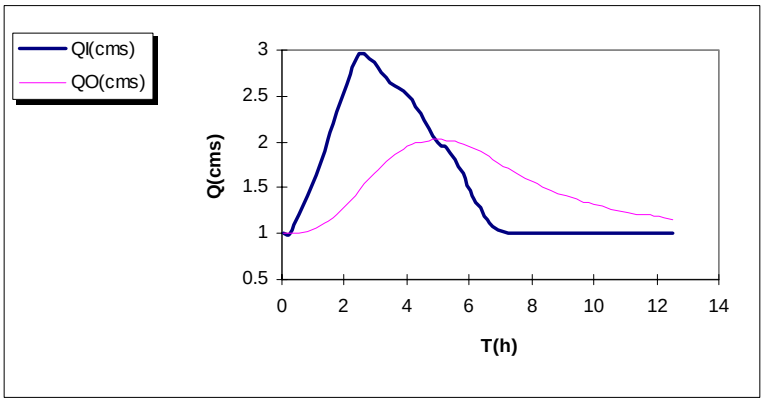

Fig.3-Flood routing calculated by model